AB FRQ questions may be found at:
http://apcentral.collegeboard.com/apc/public/repository/ap10_frq_calculus_ab.pdf
BC FRQ questions may be found at:
http://apcentral.collegeboard.com/apc/public/repository/ap10_frq_calculus_bc.pdf

**AB answers:**

#1) *f(t)* is the **rate** at which snow accumulates on the driveway, in cubic ft per hour. It is modeled by the function $f(t) = 7te^{\cos(t)}$. The **total** accumulation is the **infinite addition** of all the individual instantaneous rates of change – so we are talking about a definite integral here.

a) Total accumulation = $\int_0^6 f(t)dt = \{calc\} = 142.2746889 cu.ft.$

b) rate of change of the volume of the snow on the driveway = rate of change of snow being ADDED – rate of change of snow being REMOVED. So …
rate of change at $t = 8$ is $f(8) - g(8) = 7 \bullet 8 \bullet e^{\cos(8)} - 108 = \{calc\} = -59.58296779 cu.ft./hr$

c) total amount of snow removed = accumulation of all the instantaneous rates of change. You could use an integral, but this is really just a sophisticated algebra 2 problem. Which gives me pause – perhaps they mean to compare with the answer in part (a)? But that interpretation is definitely not clear … so – I'll use algebra 2….

$$h(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125(t-6) & \text{for } 6 \le t < 7 \\ 125 + 108(t-7) & \text{for } 7 \le t \le 9 \end{cases}$$

d) total amount of snow at 9am = total accumulation – total removed.

$$= \int_0^9 f(t)dt - \left(125 + 108(2)\right) = 367.3346064 - 125 - 216 = 26.33460637 cu.ft$$

**Possible scoring:**
(a) +1 limits, +1 integrand, +1 answer
(b) +1
(c) +1 functions, +1 "table of contents" on piecewise function
(d) +1 limits of integral, +1 subtraction of removal; +1 answer
Not certain about breakdown though …

---

#2) The table given shows 5 samples of time *t* (a particular hour) and *E(t)* (number of entries at that hour – in **hundreds**).
a) To approximate a rate of change (and we need to approximate the rate at which entries were being deposited – in hundreds) we need to use an **average** rate of change. So find the smallest interval that includes $t = 6$, and calculate the slope of the line segment joining the endpoints:

$$m = \frac{E(7) - E(5)}{7 - 5} = \frac{21 - 13}{2} = 4 \text{ hundred entries per hour}$$

AB2b) trapezoidal sum !!

$$\frac{1}{8}\int_0^8 E(t) \approx \left(\frac{1}{8}\right)[\frac{1}{2}(2-0)\left(E(2)+E(0)\right)+\frac{1}{2}(5-2)\left(E(5)+E(2)\right)+$$

$$\frac{1}{2}(7-5)\left(E(7)+E(5)\right)+\frac{1}{2}(8-7)\left(E(8)+E(7)\right)] =$$ This is the

$$\frac{1}{2}(2)(4)+\frac{1}{2}(3)(17)+\frac{1}{2}(2)(34)+\frac{1}{2}(1)(44) \Big/ 8 = \text{(but don't do this on exam)} = 85.5/8 = 10.6875$$

average number of (hundreds of entries)-hours per hour that were spent in the box.

Explanation: wow … This is the average number of hundreds of entries that were in the box from $t = 0$ to $t = 8$.

c) total number processed = accumulation of all the instantaneous rates of entries processed. So

$$total\_processed = \int_8^{12} P(t) = \int_8^{12} t^3 - 30t^2 + 298t - 976 = \{calc\} = 16(hundreds)$$

So the number left to process was the 23 hundred in the box at 8pm – 16 hundred processed = 7 hundred.

d) we want the maximum of $P(t)$ …. Solve $P'(t) = 0$ {calc} and get $t = 9.1835034$ (and also see that $P'$ is moving from positive to negative and there is a relative maximum). However, there is a relative minimum at $t = 10.816497$. So we also have to compare the value at the relative maximum with the endpoint at $t = 12$. $P(9.1835034) = 5.088662108; P(12) = 8$.

So the entries are being processed most quickly at $t = 12$ (midnight)

---

#3)

a) Total arrivals at the ride = accumulation of all the instantaneous rates of arrival. So total arrivals – integral of arrival rate (which we have graphically). So I need to find $\int_0^3 r(t)dt$ which is the **area** under the curve *r(t)*.

Area = $\frac{1}{2}(2-0)(1000+1200)+\frac{1}{2}(3-2)(1200+800) = 2200+1000 = 3200$ people.

b) The number of people waiting in line between $t = 2$ and 3 is **increasing** – The rate they are moving onto the ride is 800 people per hour, but the rate at which people are arriving at the ride is greater than 800 people an hour (from the graph). Therefore the line grows.

c) The line for the ride is longest at $t = 3$. After then, they are moved onto the ride more quickly than they arrive. Before then, the line grows. 3200-3*800=800 people added to the line that started with 700 people waiting. So the line is 1500 people long.

AB3d) Between $t = 0$ and $t = 3$, 3200 people arrive at the ride, and 3*800=2400 are moved onto the ride. So, there are 800 (3200-2400) + 700 = 1500 people standing in line at that point. You want to know when the rate of moving people onto the ride after $t = 3$ will catch up to that. So …

$1500 = \int_3^x 800 - r(t)dt$ and solve for the first $x > 3$ which makes this equation true.

---

#4) a) Area = $\int_0^9 6 - 2\sqrt{x}dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big|_0^9 = \left(54 - \frac{4}{3}9^{3/2}\right) - 0 = \{stop\_here\} = 18$

AB4b) Volume = $\int_0^9 \pi\left(7-2\sqrt{x}\right)^2 - \pi\left(7-6\right)^2\,dx$

c) $y = 2\sqrt{x} \Rightarrow \frac{y}{2} = \sqrt{x} \Rightarrow x = \frac{1}{4}y^2$ So the volume (cross sections are perpendicular to the $y$-axis – many who get this wrong will have missed that) is equal to:

$\int_0^6 \left(\frac{1}{4}y^2\right) \bullet 3 \bullet \left(\frac{1}{4}y^2\right) dy$

---

5) a) $g(3) - g(0) = \int_0^3 g'(t)dt = \frac{1}{4}\pi\left(2\right)^2 + \frac{1}{2}(1)(3) = \pi + 3/2$. So, $g(3) = 5 + \pi + 3/2$

$g(0) - g(-2) = \int_{-2}^0 g'(t)dt = \frac{1}{4}\pi\left(2\right)^2 = \pi$. So, $g(-2) = 5 - \pi$

b) On a graph of $g'$, points of inflection are indicated when the graph changes from increasing to decreasing, (corresponding to a change from concave up to concave down) or when the graph changes from decreasing to increasing (corresponding to a change from concave down to concave up). This happens on the graph given at: $x = 0$, 2, and 3.

c) $h'(x) = g'(x) - x$. The critical points of $h$ will be when $h'$ is zero or undefined, which will be for those values of $x$ where $g'(x)$ lies on the line $y = x$. There are only two points satisfying this criterion, $x = \sqrt{2}$, and $x = 3$.

At $x = \sqrt{2}$, the value of $h'$ will change from positive to negative, indicating a relative maximum at ($\sqrt{2}$, h($\sqrt{2}$)) .

At $x = 3$, the value of $h'$ will not change sign. So the point (3,h(3)) will be neither a relative maximum nor a relative minimum.

---

6) a)

$$P = (1, f(1)) = (1,2)$$
$$m = f'(1) = (1)(2)^3 = 8$$
$$SO: (y-2) = 8(x-1)$$

AB6b) Using the line in (a) and substituting $x = 1.1$, we get

$y - 2 = 8(.1) = .8 \Rightarrow y = 2.8$

This approximation will be **smaller** than the actual value of *f( 1.1 )* because both *f'* **and** *f''* are positive on the interval ( 1, 1.1 ). The function is concave up then, and the tangent line will lie below the curve.

c)

$$\frac{dy}{y^3} = xdx \Rightarrow \int \frac{dy}{y^3} = \int xdx \Rightarrow -\frac{1}{2y^2} = \frac{x^2}{2} + C$$

when $x = 1$ and $y = 2: \frac{-1}{8} = \frac{1}{2} + C \Rightarrow C = \frac{-5}{8}$

SO … $\frac{-1}{2y^2}=\frac{4x^2-5}{8}\Rightarrow\frac{-4}{4x^2-5}=y^2\Rightarrow y=\frac{2}{\sqrt{5-4x^2}}$.

---

**BC answers**

BC3) a) $$\frac{dx}{dt}=2t-4;\ \frac{dy}{dt}=te^{t-3}-1;\ \vec{v}(3)=<2,2>$$
$$speed=\sqrt{2^2+2^2}=\sqrt{8}$$

b) total distance = arc length = $\int_0^4\sqrt{(2t-4)^2+\left(te^{t-3}-1\right)^2}\,dt=\{calc\}=11.58767435$ meters

c) slope of tangent line is *dy/dx*. So …
$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{te^{t-3}-1}{2t-4}=0\Rightarrow te^{t-3}-1=0\Rightarrow t=2.20794$. At that time, *dx/dt* is positive (in fact it is positive for all $t > 2$), so the motion is to the right.

d) $t^2-4t+8=5\Rightarrow t^2-4t+3=0\Rightarrow t=1,3$.

ii) $$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{te^{t-3}-1}{2t-4}\Big|_{t=1}=\frac{e^{-2}-1}{-2}\approx-0.4323323584$$
$$\frac{te^{t-3}-1}{2t-4}\Big|_{t=3}=\frac{3-1}{2}=1$$

iii) $y(3)=\left(3+\frac{1}{e}\right)+\int_2^3 te^{t-3}-1\,dt=4$ or

$y(1)=\left(3+\frac{1}{e}\right)-\int_1^2 te^{t-3}-1\,dt=4$

---

BC5) a) x=1 y=0 m=dy/dx=1-y = 1-0 = 1
Stepsize = -0.5 delta_y= (-0.5) * m = ( -0.5 ) * 1 = -0.5
x= 1-.5 = .5 y = 0 + -0.5 = -0.5 m = dy/dx = 1-y = 1 – (-0.5) = 1.5
x = .5 - .5 = 0 y = -0.5 +stepsize*m = -0.5 + (-.5)*(1.5) = -.5 -.75 = -1.25 **

b) $\lim_{x\to1}\frac{f(x)}{x^3-1}="\frac{0}{0}"=(LH)=\lim_{x\to1}\frac{f'(x)}{3x^2}=\frac{(1-0)}{3(1)^2}=1/3$ (use dy/dx to figure out *f'(1)* )

$$\frac{dy}{1-y}=dx\Rightarrow-\ln(1-y)=x+C\Rightarrow\ln(1-y)=-x+C_1$$

c) $(1-y)=Ce^{-x}$. Since f(1)=0 ... $(1-0)=Ce^{-1}\Rightarrow C=e$

$\therefore y=1-e^{1-x}$

---

BC6) a) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... + (-1)^n \frac{x^{2n}}{(2n)!} + ...$ Remember, $n$ starts at 0 for Taylor series unless otherwise specified ….

SO,

$$\cos x - 1 = -\frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... + (-1)^{n+1} \frac{x^{2n+2}}{(2n+2)!} + ...$$

$$\frac{\cos x - 1}{x^2} = -\frac{1}{2!} + \frac{x^2}{4!} - \frac{x^4}{6!} + ... + (-1)^{n+1} \frac{x^{2n}}{(2n+2)!} + ...$$

Notice that we don't need a piecewise definition here.

b) To find relative extrema, find *f'* and set it equal to 0 (*f'*) can not be undefined. To check the critical points thus found (in reality, in a Taylor series we will only be able to find at most 1), use the second derivative test (if we have a horizontal tangent, and the function is concave up, then we have a relative minimum, and similarly for concave down and rel max).

Let *f(x)* be defined above.

$$f'(x) = \frac{2x}{4!} - \frac{4x^3}{6!} - \frac{6x^5}{8!} ... + (-1)^n \frac{(2n+2)x^{2n+1}}{(2n+4)!} + ...$$

$$f''(x) = \frac{2}{4!} - \frac{4 \bullet 3x^2}{6!} - \frac{6 \bullet 5x^4}{8!} ... + (-1)^n \frac{(2n+2)(2n+1)x^{2n}}{(2n+4)!} + ...$$

And we have $f'(0) = 0$ and $f''(0) > 0$ so 0 is a relative minimum.

c) To arrive at a Taylor series representation for $g$ we will start with that of $f$ above, and integrate term-by-term (and then add 1).

$$f(x) = -\frac{1}{2!} + \frac{x^2}{4!} - \frac{x^4}{6!} + ... + (-1)^{n+1} \frac{x^{2n}}{(2n+2)!} + ...$$

$$\int_0^x f(x)dx = -\frac{x}{2!} + \frac{x^3}{3 \bullet 4!} - \frac{x^5}{5 \bullet 6!} + \frac{x^7}{7 \bullet 8!} - \frac{x^9}{9 \bullet 10!} + ... + (-1)^{n+1} \frac{x^{2n+1}}{(2n+1)(2n+2)!} + ...$$

$$1 + \int_0^x f(x)dx = 1 + \left( -\frac{x}{2!} + \frac{x^3}{3 \bullet 4!} - \frac{x^5}{5 \bullet 6!} + \frac{x^7}{7 \bullet 8!} - \frac{x^9}{9 \bullet 10!} + ... + (-1)^{n+1} \frac{x^{2n+1}}{(2n+1)(2n+2)!} + ... \right)$$

We will keep the 1 in the beginning separate from the Taylor series ….

d)

$$g(x) = 1 + \left( -\frac{x}{2!} + \frac{x^3}{3 \bullet 4!} - \frac{x^5}{5 \bullet 6!} + \frac{x^7}{7 \bullet 8!} - \frac{x^9}{9 \bullet 10!} + ... + (-1)^{n+1} \frac{x^{2n+1}}{(2n+1)(2n+2)!} + ... \right)$$

$$T_3(x) = 1 - \frac{x}{2} + \frac{x^3}{72} \Rightarrow T_3(1) = 1 - \frac{1}{2} + \frac{1}{72}$$

As an alternating series, the error in this approximation is smaller in absolute value than the next term, which is $\frac{1}{5 \bullet 6!} < \frac{1}{6!}$